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Bangladesh Army University of Science and Technology

Department of Civil Engineering

Final Examination, Summer 2022

Course Code: CE 3231 **Level-3 Term-II**

Time: 03 (Three) hours **Full Marks: 180**

Course Title: Geotechnical Engineering II

N.B.
- The questions are of equal value.
- Figures in the margin indicate full marks allotted to each question.
- Symbols and abbreviations bear their standard meaning.
- Use a separate answer script for each PART.
- The corresponding course learning outcomes (CLOs) are given in the right most column.
- Use figures/ charts/ graphs given at the end of the question paper.

PART- A (Marks: 90)

(Answer any three questions including Q. No. 1)

			Marks	CLOs
1.	a)	Define disturbed and undisturbed samples.	(10)	1
	b)	Explain ultimate bearing capacity and allowable bearing capacity of soil.	(10)	2
	c)	The observed SPT value in a deposit of fully submerged fine silty sand was 45 at a depth of 6.5 m. The average saturated unit weight is 19.5 kN/m^3. Find the corrected SPT value for dilatancy & overburden effect.	(10)	1
2.	a)	Describe the major steps of soil exploration program.	(10)	1
	b)	A square footing shown in Fig. 1 carries a gross mass of 35000 kg column load. If the factor of safety is 3, determine the width of the footing.	(20)	2

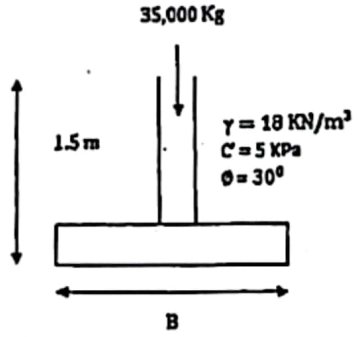


Fig. 1

3.	a)	Explain Standard Penetration Test or N value test.	(10)	1
	b)	A square foundation shown in Fig. 2 with e_L = 0.3 m and e_B = 0.15 m. Assume two-way eccentricity and determine the ultimate load of the foundation. Given that N_q = 18.4 and N_γ = 22.44 for Φ'=30°.	(20)	2

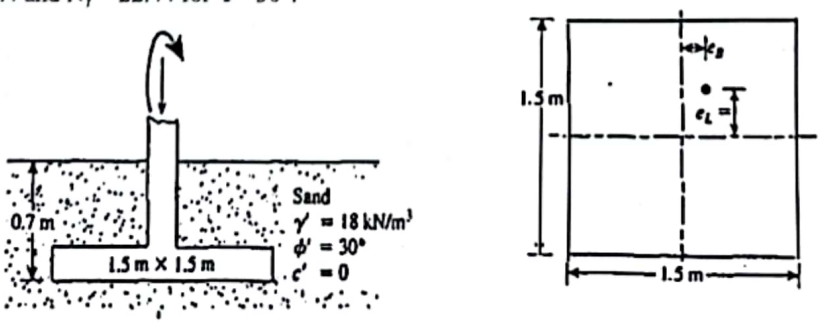


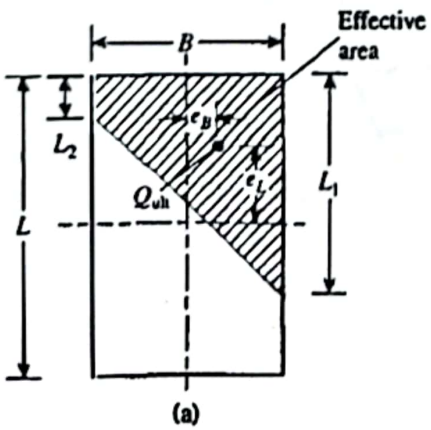
Fig. 2

4.	a)	Explain the following terms – i. End bearing pile ii. Friction pile.	(10)	2
	b)	Explain when and why pile foundation is more suitable than shallow foundation.	(10)	2
	c)	Calculate the ultimate load bearing capacity of a 25 ft long and 24" diameter pile which is embedded in clayey soil. Coefficient of adhesion = 0.5 and unconfined compressive strength of soil = 2000 psf.	(10)	2

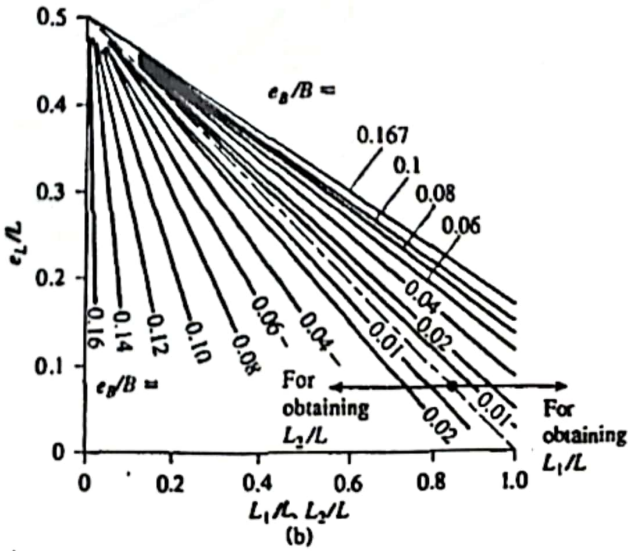
PART- B (Marks: 90)

(Answer any three questions including Q. No. 5)

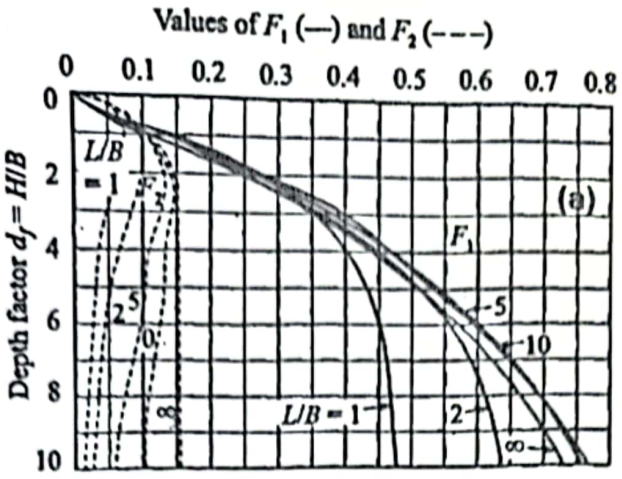
			Marks	CLOs
5.	a)	State the main causes of slope failure.	(10)	4
	b)	Determine the immediate settlement at the center of a concrete footing 1.5 x 1.5 m in size founded at a depth of 1 m in silty soil whose modulus of elasticity is 90 kg/cm^2. The footing is expected to transmit a unit pressure of 200 kN/m^2. Assume, compressive layer thickness, H=2B and μ = 0.50 Given, $I_s=0.66(D_f/B)^{-0.19}+0.025(L/B+12\mu-4.6)$	(20)	3
6.	a)	Define finite slope with examples. Sketch different parts of a finite slope.	(15)	4
	b)	A cut is to be made in a soil that has γ = 105 lb/ft^3, c = 600 lb/ft^2, and ϕ = 15°. The side of the cut slope will make an angle of 45° with the horizontal. Determine the depth of the cut slope that will have a factor of safety, FS of 3.	(15)	4
7.	a)	Define mat foundation. Explain typical pressure distribution of soil in mat foundation.	(10)	2
	b)	A cut slope was excavated in saturated clay. The slope had an angle of 40° horizontally. Slope failure occurred when the cut reached a depth of 6.1 m. Previous soil explorations showed that a rock layer was located at a depth of 9.15 m below the ground surface. Assuming an undrained condition and that γ_s=17.29 kN/m^3. i. Determine the undrained cohesion of the clay ii. Explain the nature of the critical failure circle iii. With reference to the toe of the slope, determine the distance at which the surface of sliding intersects the bottom of the excavation.	(20)	4
8.	a)	Compare between the ordinary method and Bishop's method of slice.	(10)	4
	b)	Describe the reasons for ground improvement.	(10)	4
	c)	Describe the advantages of "stone column method" over other methods of ground improvement.	(10)	4

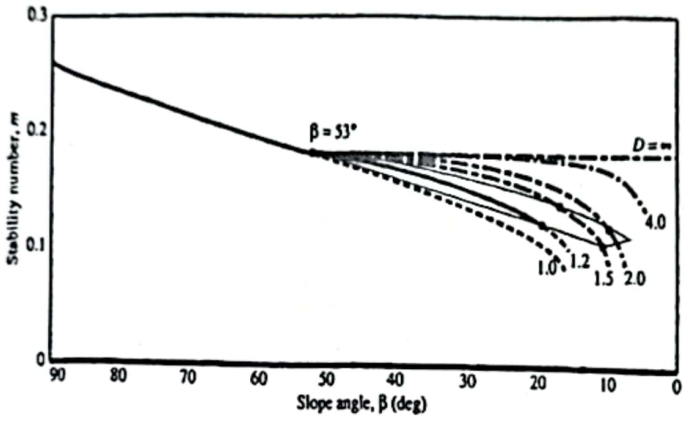


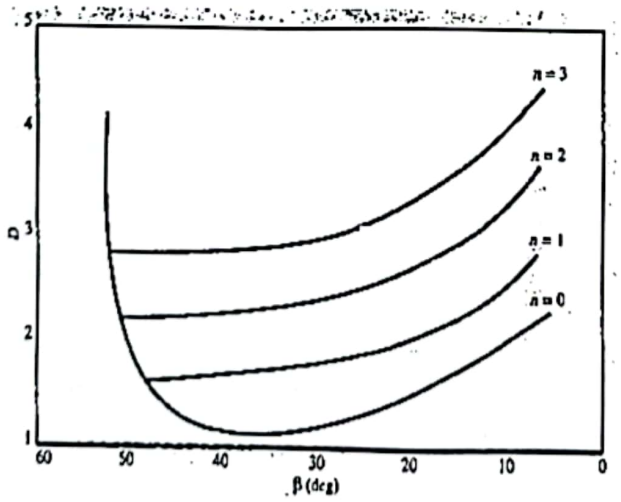
(a)



(b)


Values of F_1 (—) and F_2 (– – –)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
Depth factor $d_f = H/B$
0 2 4 6 8 10
L/B = 1
2
5
∞
(a)
F_1
5
10
L/B = 1
2
∞


Stability number, m
0.3 0.2 0.1 0
β = 53°
D = ∞
4.0
1.0 1.2 1.5 2.0
90 80 70 60 50 40 30 20 10 0
Slope angle, β (deg)


5 4 3 2 1
D
n = 3
n = 2
n = 1
n = 0
60 50 40 30 20 10 0
β (deg)

Bangladesh Army University of Science and Technology

Department of Civil Engineering

Final Examination, Summer 2022

Course Code: CE 3213 **Level-3 Term-II**

Time: 03 (Three) hours **Full Marks: 180**

Course Title: Reinforced Concrete Structures II

N.B.
- The questions are of equal value.
- Figures in the margin indicate full marks allotted to each question.
- Symbols and abbreviations bear their standard meaning.
- Use separate answer script for each PART.
- The corresponding course learning outcomes (CLOs) are given in the right most column.

PART- A (Marks: 90)

(Answer any three questions including Q. No. 1)

		Marks	CLOs
1.	A parking garage to be designed as a flat slab structure which is to carry a service live load of 220 psf. Drop panels of 8 sq. ft. will be used and each column capital consists of a 90^0 truncated cone with 48 in. diameter at the drop panel. Columns are spaced at 22 ft. on center in both sides. Material properties are f'_c = 3,000 psi, f_s = 20,000 psi. Initially, take t_1 = 9 in., t_2 = 12 in. Using the above data - i. Check the slab thickness for one-way and two-way shear. ii. Calculate the moments and steel reinforcement (for column strip only). iii. Illustrate the reinforcement detailing in a neat sketch (for column strip only).	(30)	1
2.	A floor system consisting of flat plates with edge beams is given in Fig. 1. Design a typical interior panel of this floor. Take t = 9 in. and assume it to be ok with shear check. Calculate the moments and corresponding steel area for long span only and show the reinforcement detailing. Given: Service DL = 110 psf (including self-weight) Service LL = 80 psf f'_c = 3,000 psi f_y = 60,000 psi Floor height = 12 ft. Column size = 12″ x 16″	(30)	2

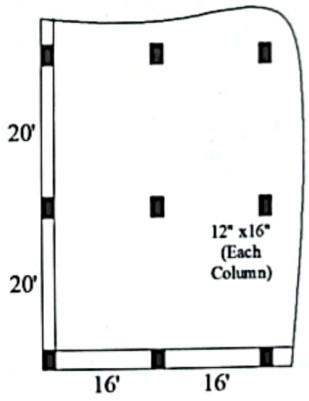


Fig. 1

3.	a)	Mention the types of shallow foundation.	(5)	2
	b)	Design an isolated single column footing using following data: Column size = 20 in. square Service DL = 250 kips Service LL = 200 kips Depth of footing = 5 ft. from EGL Unit weight of soil = 100 pcf Bearing capacity of soil q_a = 5 ksf f'_c = 4,000 psi f_y = 60,000 psi	(25)	2

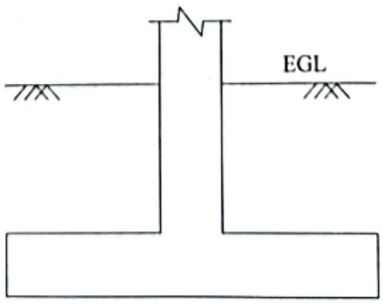


Fig. 2

4. a) Mention the general requirements of a good staircase (any five). (5) 1

b) Design any of the flights of the staircase given in Fig. 3 below. The height of riser is 6 in. Material properties are: f'_c = 4,000 psi, f_y = 60,000 psi. Assume reasonable value for any missing data. (25) 2

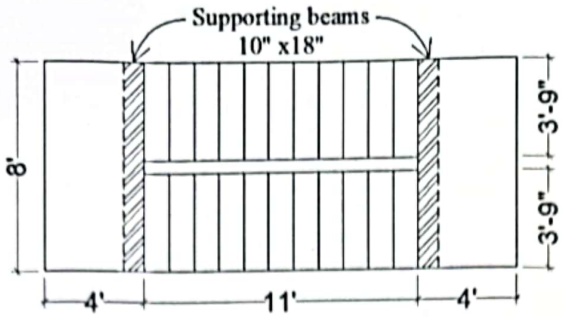


Fig. 3

PART- B (Marks: 90)

(Answer any three questions including Q. No. 5)

			Marks	COs
5.	a)	Briefly describe the behavior of ACI specified spiral column.	(5)	2
	b)	Write the advantages of the prestressed concrete over the reinforced concrete.	(10)	3
	c)	Design a circular spiral column with spiral to support P_{DL} = 475 kips and P_{LL} = 250 kips. Given, f'_c = 4 ksi and f_y = 60 ksi, steel ratio of about 3%.	(15)	2
6.	a)	Briefly describe the reasons for phi(ϕ) being low for column than beam.	(10)	2
	b)	A 14 x 22 in. column is reinforced with eight No. 9 bars as shown in Fig. 4 below. Determine the axial load P_b, moment M_b, eccentricity e_b for strong axis. Also determine P_n and M_n when e = 20 in. Given, f_c' = 4 ksi and f_y = 60 ksi.	(20)	2

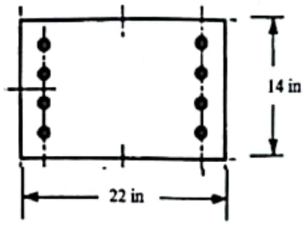


Fig. 4

7.	a)	Explain shear wall. Also, illustrate different shapes of shear walls used in structures.	(10)	3
	b)	For an exterior column, service dead load is 222 kips, maximum live load is 297 kips, dead load moment is 136 kip-ft, and live load moment is 194 kip-ft. Architectural considerations require that a rectangular column to be used, with dimensions b = 20 in and h = 25 in. Design the reinforcement for the column and design ties for it. (Necessary charts are attached with the question. Attach it with your answer script)	(20)	3
8.	a)	Explain pre-tensioned and post-tensioned prestressed concrete.	(10)	3
	b)	A prestressed member shown in Fig. 5 has a section of 8"x 12". It is eccentrically prestressed with 0.8 in^2 of high tensile steel wire which is anchored to the bulkheads at a unit stress of 150,000 psi. The centroid of the steel is 4 in above the bottom fiber. Assuming, n = 6, compute the stresses in the concrete immediately after the transfer due to the prestress only (using both exact and approximate methods).	(20)	3

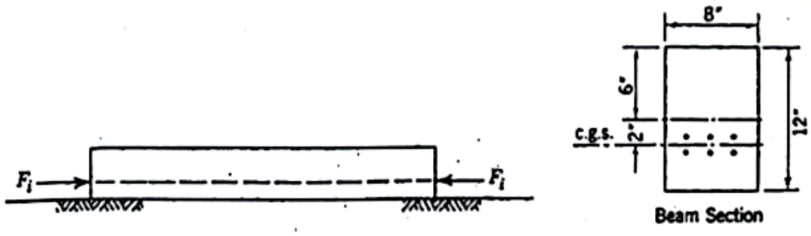


Fig. 5

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Department of Civil Engineering

Final Examination, Summer 2022

Course Code: CE 3251 **Level-3 Term-II**

Time: 03 (Three) hours **Full Marks: 180**

Course Title: Transformation Engineering I

N.B.
- The questions are of equal value.
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PART- A (Marks: 90)

(Answer any three questions including Q. No. 1)

			Marks	CLOs
1.	a)	Enumerate the concept of functional classification of highway (as per the hierarchy of movement) with the help of a diagram.	(10)	1
	b)	Explain the main causes of traffic congestion in Bangladesh. Put forward your suggestions for reducing this problem.	(10)	2
	c)	State are the objectives of traffic studies. List the causes of traffic accident and suggest the remedial measures.	(10)	2
2.	a)	Describe the general requirements of traffic control devices. Compare road marking with road sign.	(10)	1
	b)	Explain how grade (up or down grade), weather condition (dry or wet), speed of vehicle and height of the driver affect stopping sight distance (SSD) and passing sight distance (PSD)?	(10)	2
	c)	Estimate the total passing sight distance for a two-lane highway for average passing speed of 62 mph. The initial maneuver time is 4.5 sec and average acceleration is 1.50 mph/sec. Consider the time of passing vehicle occupying right lane as 13.3 sec and clearance length as 300 ft.	(10)	2
3.	a)	How can the shoulder and median improve roadway safety? Explain.	(10)	2
	b)	Write short note: (i) Stop sign and yield sign (ii) Retroreflective sign and marking (iii) Pedestrian scramble.	(15)	1
	c)	Enumerate the basic elements of transportation planning with help of a flow chart.	(5)	1
4.	a)	Draw the schematic diagram showing the direction of traffic flow of Trumpet and Diamond type of interchanges.	(10)	1
	b)	State the methods of attaining superelevation in a highway. Explain any one method with the help of a sketch.	(15)	2
	c)	Explain how drivers, vehicles and traffic characteristics influence highway design.	(5)	1

PART- B (Marks: 90)

(Answer any three questions including Q. No. 5)

			Marks	CLOs
5.	a)	Elaborate the terms a) AADT b) ADT c) HEF d) PCE f) PCU.	(10)	1
	b)	An Engineer needs to determine the AADT of a roadway section for which volume distribution is given in Table 1, 2, 3. The engineer collected the following data during a survey on Tuesday in the month of MAY. Determine the AADT.	(20)	3

Hour	Volume
7:00-8:00 a.m.	400
8:00-9:00 a.m.	535
9:00-10:00 a.m.	650
10:00-11:00 a.m.	710
11:00-12:00 p.m.	650

6. a) Define 'Origin-Destination' survey. Show different kinds of traffic in Saidpur area by a diagram. (10) 1

b) (i) Following travel time was recorded for 4 vehicles traversing a 1 mile segment of a highway. (10+10) 3

Vehicle	1	2	3	4
Time (min)	1.6	1.2	1.5	1.7

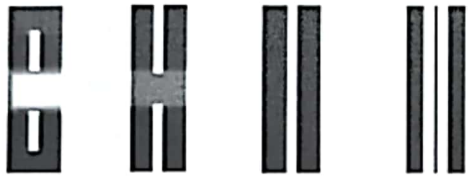
Determine space and time mean speeds.

(ii) Redraw the plot given in Fig. 1 and calculate the 'Pace'.

7. a) Saidpur Cant. Road connects to the 'Rangpur Road' near Saidpur Terminal. It is a very accident-prone area because of sudden entrance of vehicles at the main Highway. Describe with figures how Informatory Signs (Stop and Yield) can solve the problem. (10) 1

b) Explain the rules and regulations of 'Placement of Traffic Sign' and 'Hight of Traffic Signs' at the highways. (20) 1

Describe the traffic markings given in Fig. 2.



(a) (b) (c) (d)

Fig. 2

8. a) Assume that we know there are 1,000 trips being made between zones Rangpur and Saidpur (which we obtained from trip distribution). There are three modes available to make this trip. The utility of the individual modes is defined as (10) 1

$$U_{auto} = 1.0 - 0.1(TT_{auto}) - 0.05(TC_{auto})$$
$$U_{bus} = -0.1(TT_{bus}) - 0.05(TC_{bus})$$
$$U_{walk} = -5.0 - 0.1(TT_{walk})$$

where TT = travel time by mode in minutes
TC = travel cost by mode in dollars

Assume that we know that the travel time for auto is 5 minutes, for bus 15 minutes, and for walking 20 minutes. The corresponding costs are \$0.60 for auto and \$0.50 for bus.

Find the probabilities associated with the use of each mode using 'Logit Model'.

b) The area near BAUST have been developed in recent years. Consequently, the flow of traffic increased remarkably on the road in front of BAUST. (20) 2

(i) Name the organizations who play an important role in maintaining the roads in this area.

(ii) Define the development of the area by 'Land Use' diagram.

(iii) Comment on the factors related to the Highway Safety in the mentioned area.

Table 1 Hourly Expansion Factors for a Rural Primary Road

Hour	Vol.	HEF	Hour	Vol.	HEF
6:00-7:00 a.m.	294	42.01	6:00-7:00 p.m.	743	16.6
7:00-8:00 a.m.	426	28.99	7:00-8:00 p.m.	706	17.5
8:00-9:00 a.m.	560	22.05	8:00-9:00 p.m.	606	20.4
9:00-10:00 a.m.	657	18.8	9:00-10:00 p.m.	489	25.3
10:00-11:00 a.m.	722	17.11	10:00-11:00 p.m.	396	31.2
11:00-12:00 p.m.	667	18.52	11:00-12:00 a.m.	360	34.3
12:00-1:00 p.m.	660	18.71	12:00-1:00 a.m.	241	51.2
1:00-2:00 p.m.	739	16.71	1:00-2:00 a.m.	150	82.3
2:00-3:00 p.m.	832	14.84	2:00-3:00 a.m.	100	124
3:00-4:00 p.m.	836	14.77	3:00-4:00 a.m.	90	137
4:00-5:00 p.m.	961	12.85	4:00-5:00 a.m.	86	144
5:00-6:00 p.m.	892	13.85	5:00-6:00 a.m.	137	90.2
		Total daily volume =	12350		

For question 5b

Table 2 Daily Expansion Factors for a Rural Primary Road

Day of Week	Volume	DEF
Sunday	7,895	9.515
Monday	10,714	7.012
Tuesday	9,722	7.727
Wednesday	11,413	6.582
Thusrday	10,714	7.012
Friday	13,125	5.724
Saturday	11,539	6.51
	Total weekly volume =	75,122

For question 5b

Table 3 Monthly Expansion Factors for a Rural Primary Road

Day of Week	ADT	MEF
January	1350	1.756
February	1200	1.976
March	1450	1.635
April	1600	1.482
May	1700	1.395
June	2500	0.948
July	4100	0.578
August	4550	0.521
September	3750	0.632
October	2500	0.948
November	2000	1.186
December	1750	1.355
	Total yearly ADT volume =	28450
	AADT =	2371

For question 5b

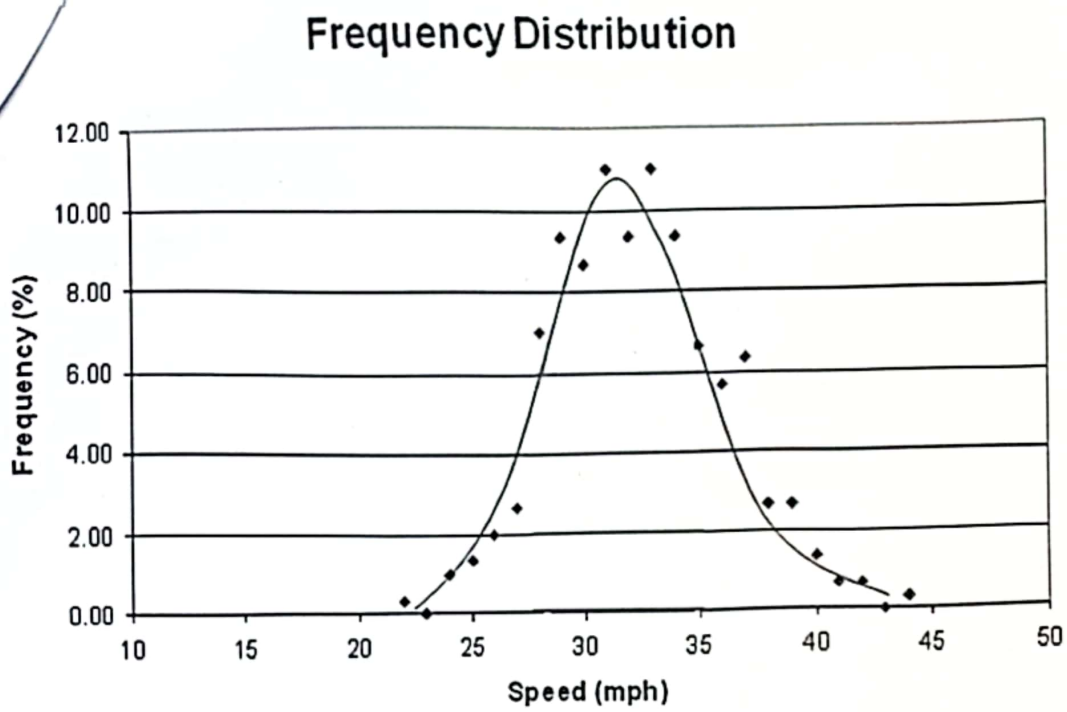


Fig. 1

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Bangladesh Army University of Science and Technology

Department of Civil Engineering

Final Examination, Summer 2022

Course Code: CE 3261 **Level-3 Term-II**

Time: 03 (Three) hours **Full Marks: 180**

Course Title: Water Resources Engineering I

N.B.
- The questions are of equal value.
- Figures in the margin indicate full marks allotted to each question.
- Symbols and abbreviations bear their standard meaning.
- Use separate answer script for each PART.
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PART- A (Marks: 90)

(Answer any three questions including Q. No. 1)

			Marks	CLOs
1.	a)	Explain the following terms (any two) i) Surface runoff ii) Cloud seeding iii) Residence time	(5)	1
	b)	Describe the different forms of precipitation.	(10)	2
	c)	An urban area has a runoff coefficient of 0.30 and an area of 0.85 km^2. The slope of the catchment is 0.006 and the maximum length of travel of water is 950 m. The maximum depths of rainfall along with durations for a 25-year return period are given below:	(15)	3

Duration (min)	5	10	20	30	40	60
Depth of rainfall (mm)	17	26	40	50	57	62

Estimate the required peak flow rate for the catchment for a 25-year return period.

2.	a)	Explain hydrology and write down the practical application of hydrology.	(10)	1
	b)	Describe hydrologic cycle with necessary illustration and state the processes.	(20)	1
3.	a)	Explain natural and artificial channel.	(5)	1
	b)	Briefly describe different types of streams.	(15)	2
	c)	A trapezoidal stream has a bottom width of 6 m and a side slope of 2:1. Compute the discharge and determine the state of flow in this channel if the depth of flow is 1.5 m and the mean velocity is 2.3 m/s.	(10)	2
4.	a)	Explain Thiessen polygon method for the calculation of average rainfall.	(15)	3
	b)	At a climate station, air pressure is measured as 100 kPa, air temperature as 20°C and dew point temperature is 16°C. Determine the corresponding vapor pressure and relative humidity.	(15)	2

PART- B (Marks: 90)

(Answer any three questions including Q. No. 5)

			Marks	CLOs
5.	a)	An area near Saidpur Airport gets flooded frequently. The authority took decision to do soil improvement in the area. For which they need to assess the infiltration rate and infiltration indices of the ground from rainfall data. The data found was given below:	(10)	1

Time (hr)	0	2	4	6	8	10	12	14	16
Cumulative Rainfall (cm)	0	0.4	1.3	2.8	5.1	6.9	8.5	9.5	1

Suggest the factors that need to be in mind of the assessment officer while calculating the infiltration.

b) From the Data of question 5(a) identify the Φ index if the run off is 5.8 cm. (20) 2

6. a) (i) Draw a typical figure of a 'Hydrograph' and identify different components. (10) 1

(ii) Define a 'Unit Hydrograph'

b) Rainfall of magnitude 3.8 cm and 2.8 cm occurring on two consecutive 4h durations on a catchment of area 27 km^2 produced the following hydrograph at the outlet of the catchment. Estimate the 'Rainfall Excess'. (20) 3

Time (hr)	0	6	12	18	24	30	36	42	48	54	60
Flow (m^3/sec)	5	13	26	21	16	12	9	7	5	5	4.5

7. a) In a catchment area, the average rainfall needs to be calculated. For which the location of the stations needs to be changed frequently. Comment on the suitable method for determining the rainfall average of the area based on comparison among three methods. (10) 1

b) From the Fig. 1 (attached with question) calculate the average rainfall by 'Thiessen Polygon' method. The rainfall depth of each gage is given: (20) 3

Station	P1	P2	P3	P4	P5
Observed rainfall (mm)	10	20	30	40	50

(N.B.: Attach the Fig. 1 with the answer sheet.)

8. a) Write five Uses of Remote Sensing in Hydrology. (10) 1

b) Given the ordinates of a 4h unit hydrograph as below. Derive the ordinates of 12h Unit Hydrograph for the same catchment. (20) 3

Time(hr)	0	4	8	12	16	20	24	28	32	36	40
Ordinates of 4hr UH	0	20	80	130	150	130	90	52	27	15	5

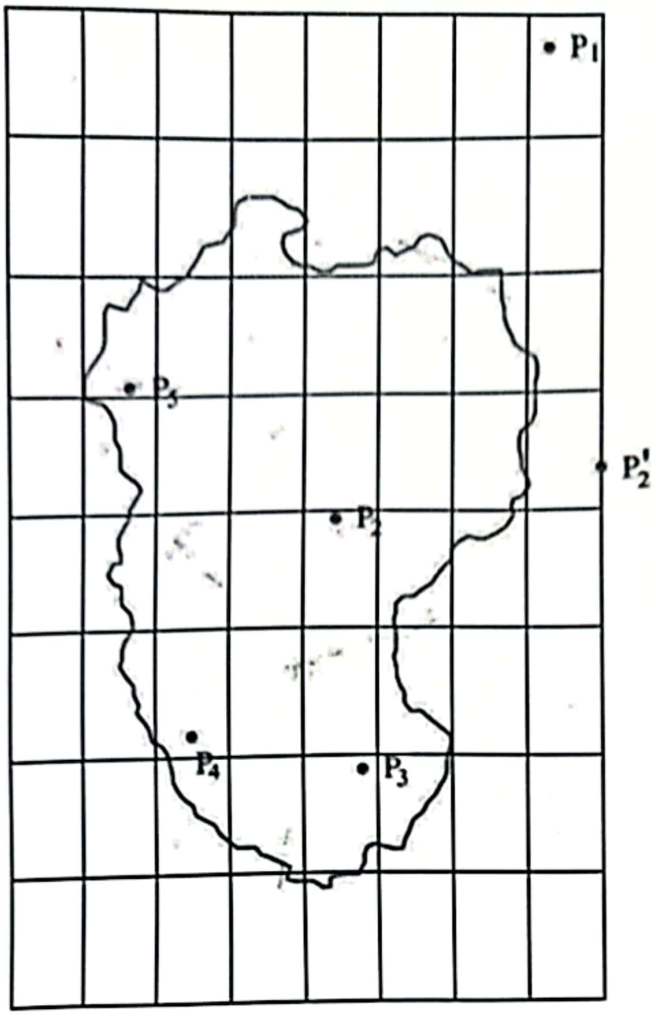

Fig. 1

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Bangladesh Army University of Science and Technology

Department of Civil Engineering

Final Examination, Summer 2022

Course Code: CE 3211 **Level-3 Term-II**

Time: 03 (Three) hours **Full Marks: 180**

Course Title: Structural Analysis and Design II

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- The questions are of equal value.
- Figures in the margin indicate full marks allotted to each question.
- Symbols and abbreviations bear their standard meaning.
- Use separate answer script for each PART.
- The corresponding course learning outcomes (CLOs) are given in the right most column.

PART- A (Marks: 90)

(Answer any three questions including Q. No. 1)

			Marks	CLOs
1.	a)	For a 6 storied moment resisting RCC building, calculate the seismic load on each floor level using the following data: Height of ground floor = 4 m — DL on ground & top floor = 2500 kN/m^2 Height of other five floors = 3 m — DL on 1st to 4th floor = 3000 kN/m^2 Plan area = 15m x 20m — z = 0.25, I = 1.25, R = 10, s = 1.50	(20)	1
	b)	Describe briefly, the effect of the factors that are used in the analysis of buildings for wind load.	(10)	1
2.		Solve the following frame by Factor method and find out the moments at the end of each column and beam. Also draw the bending moment diagram and qualitative deflected shape for it. Assume, all the columns have the equal dimensions.	(30)	2

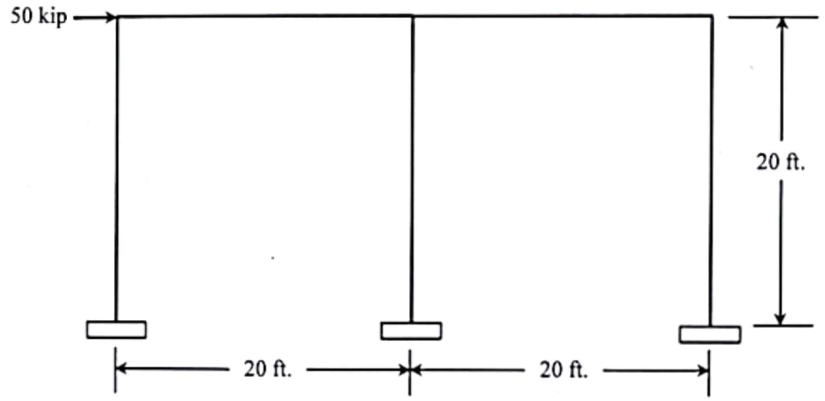


Fig. 1

3.	a)	State the assumptions for analyzing a building frame by cantilever method.	(5)	1
	b)	For the frame shown in Fig. 2, calculate the relevant forces at each joints by cantilever method. Given, F_1 = 10 kips and F_2 = 20 kips. Assume, all the columns have the equal dimensions.	(25)	1

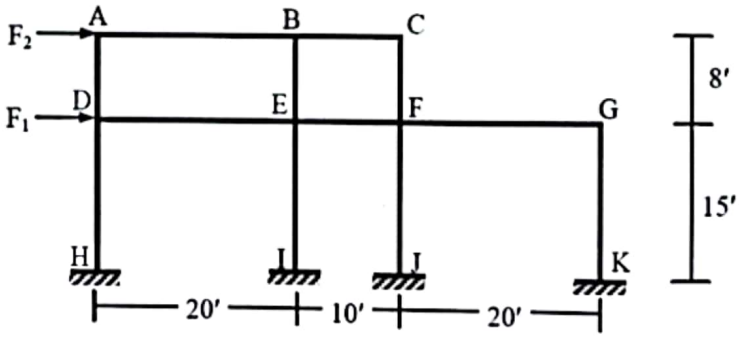


Fig. 2

4. a) State the assumptions for analyzing a building frame for gravity load. (5) 1

b) Analyze the frame structure loaded as shown in Fig. 3 using the approximate location of hinges to draw the shear force and bending moment diagrams of the beams and columns. (25) 1

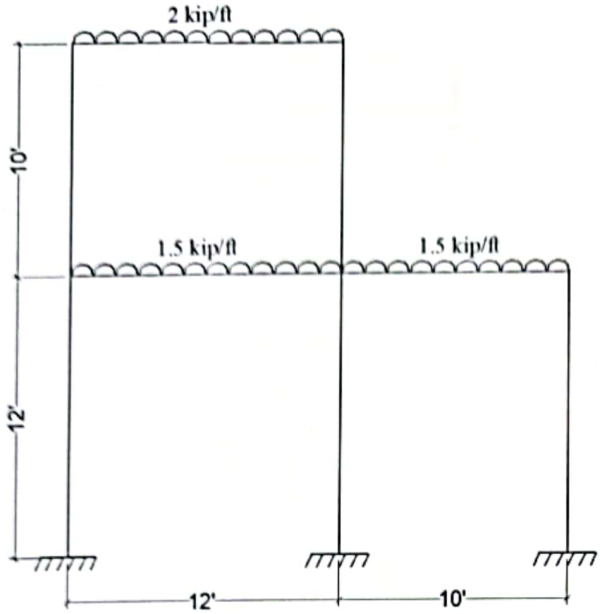


Fig. 3

PART- B (Marks: 90)

(Answer any three questions including Q. No. 5)

		Marks	CLOs
5. a)	Explain Degree of Static Indeterminacy (DOSI) & Degree of Kinematic Indeterminacy (DOKI).	(10)	2
b)	A simply supported beam is subjected to a uniform distributed load over the length of the beam in Fig. 4 below. Determine the vertical deflection at midspan of the beam using Virtual Work Method. Assume, EI to be constant.	(20)	3

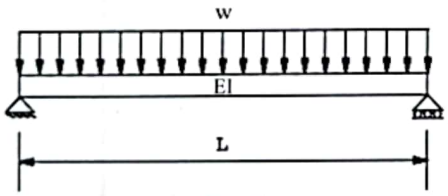


Fig. 4

6. a)	A truss is subjected to a vertical load as given in Fig.5. Determine the vertical displacement of point B using virtual work method due to external load at B & D. Given, E = 30000 *ksi* , A=100 in^2	(20)	3

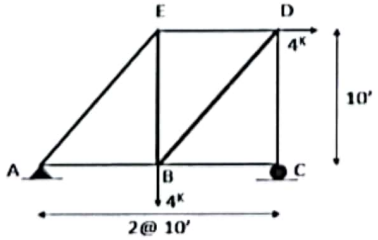


Fig. 5

b)	ii) Determine vertical displacement of point B in Fig.5 due to a decrease of temperature of 50 °F at the bottom chord. Given, E = 30000 *ksi* and $\alpha_T = (1/150000)$ /°F	(10)	3

7. a) Sketch bending moment diagram of the beam shown in Fig. 6 (a). Use consistent deformation method. Ignore axial deformation. (20) 2

Given, P = 10 kN, L = 10m, EI = 1000 kN-m^2.

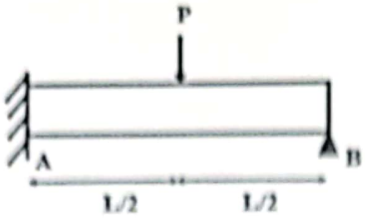


Fig. 6(a)

b) Determine the Degree of Static Indeterminacy (DOSI) of Fig. 6 (b) given below. (10) 2

Fig. 6(b)

8. a) Define space truss. Compare between space truss and plane truss. (10) 2

b) Calculate the reaction force of the support R in Fig. 7. Use consistent deformation method. Ignore axial deformation. (20) 2

Given, L= 10ft , P = 10^k, E= 30000 ksi , A = 100 in^2, I = 1000 in^4.

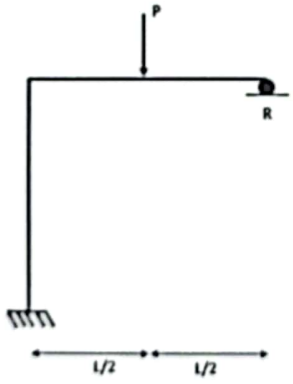
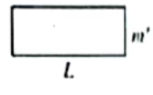
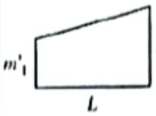
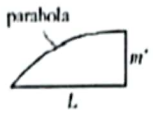
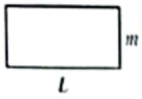
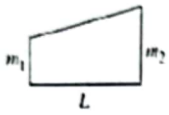
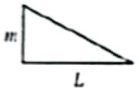


Fig. 7

Table for Evaluating $\int_0^L m\, m'\, dx$

$\int_0^L m\, m'\, dx$				
	$mm'L$	$\frac{1}{2}mm'L$	$\frac{1}{2}m(m'_1 + m'_2)L$	$\frac{2}{3}mm'L$
	$\frac{1}{2}mm'L$	$\frac{1}{3}mm'L$	$\frac{1}{6}m(m'_1 + 2m'_2)L$	$\frac{5}{12}mm'L$
	$\frac{1}{2}m'(m_1 + m_2)L$	$\frac{1}{6}m'(m_1 + 2m_2)L$	$\frac{1}{6}[m'_1(2m_1 + m_2) + m'_2(m_1 + 2m_2)]L$	$\frac{1}{12}[m'(3m_1 + 5m_2)]L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'(L + a)$	$\frac{1}{6}m[m'_1(L + b) + m_2(L + a)]$	$\frac{1}{12}mm'\left(3 + \frac{3a}{L} - \frac{a^2}{L^2}\right)L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'L$	$\frac{1}{6}m(2m'_1 + m'_2)L$	$\frac{1}{4}mm'L$